CONTEXTS FOR LEARNING mathematics

Minilessons for Extending Addition and Subtraction

A Yearlong Resource

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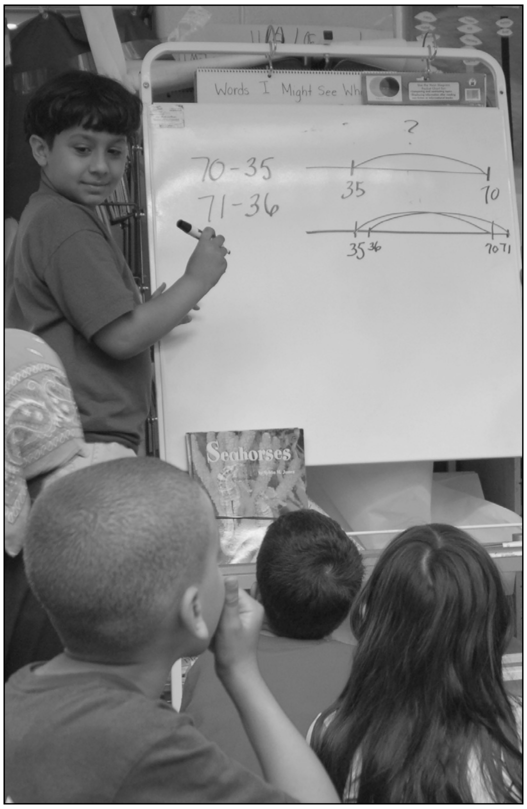
Overview



Unlike many of the other units in the *Contexts for Learning Mathematics* series, which consist of two-week sequences of investigations and related minilessons, this unit is meant to be used as a resource of 68 minilessons that you can choose from throughout the year. In contrast to investigations, which constitute the heart of the math workshop, the minilesson is more guided and more explicit, designed to be used at the start of math workshop and to last for ten to fifteen minutes. Each day, no matter what other unit or materials you are using, you might choose a minilesson from this resource to help your students develop efficient computation. You can also use them with small groups of children as you differentiate instruction.

This guide is structured progressively, moving from the use of counting and place value activities, to the use of a connecting cube train (as a number line) allowing for counting if needed but where the five- and ten-structures are emphasized, to the open number line where mental jumps of numbers being added or subtracted are recorded. The last section supports the use of coins to establish helpful landmark numbers. Although you may not use every minilesson in this resource, you will want to work through it with a developmental progression in mind.

Except for the first section, which is made up of counting and place value activities, the bulk of the minilessons in this resource unit are crafted as groups, or "strings," of tightly structured computation problems designed to encourage children to look to the numbers first, before they decide on a computation strategy. The strings are crafted to support the development of a variety of mental math strategies as well as the traditional algorithms. These strings are likely to generate discussion on certain strategies or big ideas underlying an understanding of addition and subtraction.

The Mathematical Landscape: Developing Numeracy

Try an experiment. Calculate 3996 + 4246. Don't read on until you have an answer. If you are like most people who are products of the American school system, you probably got a pencil and paper, wrote the numbers in columns, added the units and regrouped, then added each remaining column, right to left. You used the regrouping algorithm—the procedures you were taught in school. If you didn't, congratulations! You probably have number sense.

What would someone with good number sense do? What would a mathematician do? Both would look to the numbers first to decide on a strategy. Because 3996 is so close to the more friendly number 4000, a more efficient strategy would be to remove 4 from 4246 resulting in 4242. Combining the 4 with 3996 establishes an equivalent form of the problem, 4000 + 4242, which is easy to calculate mentally. But what if the numbers can't be made friendly so easily? Try 234 + 136. You could use 235 + 135, or 240 + 130: the answer is 370. Or try 289 + 79. You could use 290 + 80 − 2, or maybe 300 + 80 − 11 − 1: the answer is 368. If you try to find numbers that can't be made friendly—numbers where the algorithm (the regrouping procedure) is faster—you will probably discover that the only time the regrouping algorithm is the best strategy is when you are calculating long columns of numbers. And today when we have many large numbers to add, we use a calculator.

How about subtraction? Let's try 3400 − 189. Do you need pencil and paper? You could just add 11 to each number; the equivalent result, 3411 − 200, can be done mentally quite easily. Consider the age difference between a 71-year-old and a 36-year-old. A year ago they were 70 and 35, but the difference was the same (71 − 36 = 70 − 35). Any subtraction problem can be made friendlier just by thinking about subtraction as the difference between two numbers on a number line. Just slide the numbers back and forth, while keeping the difference constant, until you reach a nice landmark number that makes the subtraction easy.

Regrouping can be a challenge with 3400 − 189 because of the zeroes — at least it is a challenge for children, as researchers have well documented. If they attempt to regroup, they often make many place value mistakes and lose sight of the actual numbers, arriving at quite unreasonable answers. Yet, because they trust in the algorithm, they assume they are correct. Many children end up with 3389 because they take 0 away from 89. They don't stop to wonder how taking away almost 200 could result in a number almost identical to what they started with. They trust their understanding of the algorithm, rather than making an assessment of reasonableness.

To be successful in today's world, we need a deep conceptual understanding of mathematics. We are bombarded with numbers, statistics, advertisements, and similar data every day—on the Internet, on the radio, on television, and in newspapers. We need good mental ability and good number sense in order to evaluate advertising claims, estimate quantities, efficiently calculate the numbers we deal with every day and judge whether these calculations are reasonable, add up restaurant checks and determine equal shares, interpret data and statistics, and so on. We need a deep understanding of number and operation that allows us to both estimate and make exact calculations mentally. This understanding includes algorithms, but it places emphasis on mental arithmetic and a repertoire of strategies.

Depending on the numbers, the algorithm is often slow. It only seems faster to most adults because they have always used algorithms. The procedures have become habits that require little thinking. Calculating with number sense as a mathematician means having many strategies at your disposal and looking to the numbers first *before* choosing a strategy. How do we, as teachers, develop children's ability to do this? How do we engage them in learning to be young mathematicians at work?

Using Minilessons to Develop Number Sense: An Example

Minilessons are usually done with the whole class together in a meeting area. Young children often sit on a rug; for older students, benches or chairs can be placed in a U-shape. Clustering students together like this, near a chalkboard, is helpful because you will want to provide an opportunity for pair talk at times, and you will need space to represent the strategies that will become the focus of discussion. The problems are written one at a time and learners are asked to determine an answer. Although the emphasis is on the development of mental arithmetic strategies, this does not mean learners have to solve the problems *in* their heads—but it is important for them to do the problems *with* their heads! In other words, encourage children to examine the numbers in the problem and think about clever, efficient ways to reach a solution. The relationships among the problems in the minilesson will support children in doing this. By developing a repertoire of strategies, an understanding of the big ideas underlying why they work, and a variety of ways to model the relations, children are developing powerful toolboxes for flexible and efficient computation. Enter a classroom with us and see how this is done.

"I broke the 15 into a 10 and a 4 and a 1."Brittani, a second grader in New York City, is explaining how she solved the problem 15 + 9."Then I gave the 1 to the 9, that made 10 . . . and I knew that 10 plus 10 was 20, and 4 more made 24."

Brittani's teacher, Jennifer, has chosen the numbers in her string to encourage children to make use of 10 when they add. Ten can be helpful in several ways:

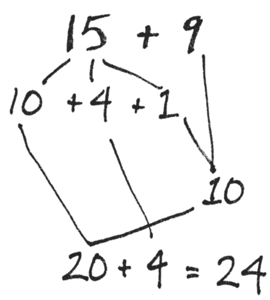
- By taking leaps of ten all at once and adjusting—for example, 15 + 9 = 15 + 10 − 1.
- By moving to the next multiple of 10—for example, 15 + 9 = 15 + 5 (to get 20, the next multiple of 10) + 4.
- By using compensation to make a problem with 10 in it—for example, 15 + 9 = 14 + 10.

Jennifer began with 15 + 10 and then moved to 15 + 9, because the number 9 is so close to 10. She anticipated that some children, noticing that, would make a leap of ten and subtract 1, effectively bringing the strategy up for discussion. The third problem in her string is 15 + 19. This addend (19) is 10 more than the 9 in the second problem. Here Jennifer anticipates that whatever strategy her students have found effective in the second problem will be extended in the third. If children simply count, without using 10, patterns will still appear in the answers that are likely to engender discussion. Her fourth problem (28 + 19) will challenge the children further. Will they use an equivalent expression: 28 + 20 − 1 or 30 + 17? The next problem (28 + 32) is similar; will they do 28 + 30 + 2 or make the problem 30 + 30? In the last problem (39 + 21), she hopes that even children who were not initially making use of the tens will do so here, after the discussion, since 39 is so close to 40, and 21 is so close to 20. She anticipates that many children now will see how easy it is to use an equivalent form, 40 + 20.

Although Jennifer had thought about the problems beforehand and had the string she is using ready, she does not put all the problems on the board at once. Instead, she writes one at a time, and children discuss their strategies before the subsequent problem is presented. This way, the children can consider the strategies from the previous problem as well as the numbers, and they are prompted to think about the relationships of the problems in the string as they go along. Sometimes, depending on the strategies she hears, Jennifer adjusts the problems in her planned string on the spot to ensure that the strategies she is attempting to develop are discussed and tried out. For example, if none of the children use ten when they solve 15 + 9, Jennifer might insert two more problems that use ten explicitly—27 + 10, 32 + 10—and then return to the 9 with a problem like 32 + 9.

Brittani is beginning to make tens, but to do so she writes the 15 as three addends (10 + 4 + 1)—a slower, more cumbersome strategy. For just that reason, Jennifer begins the discussion with this strategy and uses it as a scaffold for more efficient strategies.

After Brittani shares her strategy, Jennifer paraphrases, drawing three short lines from the number 15 to represent the split. "So you broke up the 15 into 10, 4, and 1." She writes these numbers under the 15 as in the figure below:



"Then you made another 10 with the 1 and the 9." She draws lines connecting these numbers. "And then you added the tens to make 20, and the 4 left made 24?"

By representing children's strategies, Jennifer provides a written record of the action. This allows other children to "see" the strategy; it becomes a picture that can be discussed. Purely verbal explanations are often too difficult for children to understand, particularly when the strategy being described is different from the one they used. Having a representation of the action allows more children to understand and to take part in the discussion.

"Any questions for Brittani? Did anybody do it a different way? Luke?"

"I used the hundred chart," says Luke, pointing to the large pocket chart containing the numbers 1 through 100, that hangs next to the chalkboard, "and I took 1 away."

Although Jennifer's goal is to *develop* mental math strategies, that does not mean that during the minilesson children must *use* mental strategies. Although Brittani has solved the computation mentally, Luke has not.

"Show us, Luke," Jennifer says.

"I started with 15, and I jumped down a row to 25. Then I took 1 away." Luke points to the numbers on the chart as he explains.

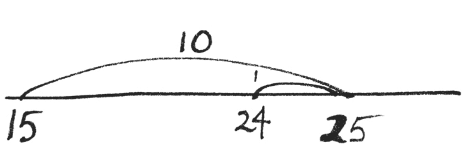
Although Jennifer knows why he took 1 away, she is aware that this is a difficult leap in thinking for many of her beginning second graders. So she asks Luke to elaborate: "Tell us why you took 1 away."

Although Luke understands why he took 1 away, he struggles to communicate his reasoning to his classmates: "Because I knew 15 plus 10 was 25, but I needed to take 1 away." Other children begin to question him.

"But why, Luke?" Katia asks, puzzled. "How did you know to take 1 away?"

This time Luke's response is clearer: "Because I only needed 9, not 10." Several children, including Katia and Brittani, now nod with understanding and agreement. Although Luke has not solved the problem mentally — he has used the hundred chart as a tool — his strategy is very efficient. He has kept the 15 whole and made use of the fact that 9 is close to 10. He takes a leap of 10 all at once and subtracts 1 at the end. Because the hundred chart is structured in tens, children often notice the patterns and begin to move vertically on it rather than only horizontally. When they move vertically, they are taking leaps of ten; when they move horizontally, they are counting by ones. Although they notice and use the patterns on the hundred chart, they are not necessarily thinking of leaps of ten mentally, however. This is an important landmark strategy — one that Jennifer wants to highlight in this minilesson.

To do so, she models Luke's strategy on the open number line:



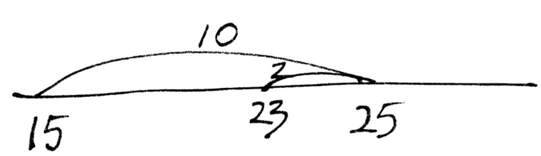
She wants Luke to think in leaps of ten, mentally, on a number line without needing the hundred chart as a manipulative. By connecting the hundred chart to the open number line, she attempts to help all the children move from a tool—a model of their thinking—to a more formal mathematical model *for* thinking—a number line.

"So you took a leap of ten," she paraphrases as she draws the leap on the number line and writes 10 above it and 25 under it. "And then you went back 1 because you only needed to add 9." Jennifer writes 1 and marks a small leap backward, completing the representation by writing 24. "Wow, that's a neat strategy, isn't it?" she finishes, inviting the class to reflect on Luke's strategy.

Although the next problem in the string that Jennifer has prepared is 15 + 19, she decides to try 15 + 8 instead. She wants to see whether children will make use of Luke's strategy. Underneath the last problem, which now reads 15 + 9 = 24, she writes 15 + 8. "Show me with thumbs-up when you have an answer." After giving an appropriate amount of think time, which she gauges by looking at the number of thumbs up, she starts a discussion. "Tyler?"

"It's 23. Because like Luke said, he jumped down 10 and went back 1. That was 24. So I jumped back 2."

Jennifer draws another open number line to represent Tyler's thinking:



"Neat, you used Luke's idea." Luke smiles broadly. "How many of you did it that way?" Several hands go up. "Did anybody do it a different way? Karen?"

Although it seems a simple step to go from 15 + 9 = 24 to 15 + 8 = 24 − 1, and many children do see this relationship, Karen does not. She sees it as a separate problem, and she needs to make sense of it for herself.

"Can I use the string of cubes?" Karen points to the 50 connecting cubes (in groups of 5 in alternating colors) on a wire, which Jennifer has made and strung across the top of the chalkboard.

"Sure," Jennifer says. "What do you want me to push over?"

"Fifteen," Karen responds tentatively.

"OK, so here's 10." Jennifer deliberately slides over two groups of 5 in order to reinforce children's understanding, and use, of ten. "So how many more do I need?" Will Karen know immediately that 5 more are needed, or will she have to count?

"You need 5 more," Karen responds.

"OK." Jennifer moves them over. "And what do you want to add?"

"Add 8."

"OK, here's 10 more." Jennifer moves over two groups of 5, again attempting to support the development of strategies based on tens. "How can I make this into 8?"

"Take 2 off," Karen answers, and Jennifer moves 2 cubes back to the right.

"If we started with 15, how many more would we need to get 20?" Jennifer points to the cubes as she attempts to stretch Karen's thinking toward compensation.

"Five more."

"Okay," Jennifer agrees, and shifts 5 cubes over to the left, making 20 and 3. "So we know it's 23?"

Although children may initially compute problems in a string in various ways, as they share their strategies they notice and discuss patterns in the string and in the answers. They become intrigued by the fact that answers are the same, or that they are different by one or by ten, and they want to investigate why these patterns are occurring. They are impressed by, and interested in, their classmates' strategies and often adopt them when the strategies seem appropriate and more efficient. These young mathematicians are composing and decomposing numbers flexibly as they add. They are inventing their own strategies. They are looking for relationships among the problems. They are looking at the numbers first before they decide on a strategy.

Children don't do this automatically. Jennifer has worked to develop this ability in her students by focusing on computation every day, during minilessons with strings of related problems. She has developed the big ideas and models through investigations, but once this understanding has been constructed, she promotes fluency with computation strategies in minilessons such as this one.

Using Models during Minilessons

An important component of the mathematical landscape is the development of mathematical modeling. In her minilesson, Jennifer makes use of several tools, representations, and models. She uses short lines to represent splitting. She uses the open number line to represent other strategies. Children go back and forth from the hundred chart to the string of connecting cubes to the open number line. While the hundred chart is likely to encourage the use of leaps of ten, because children can make use of columns, the train of connecting cubes in alternating groups of five is more likely to support use of the five-structure. But because the hundred chart and the cubes allow children to count by ones, their use can lead to a reading-off approach, rather than promoting the ability to calculate mentally. In contrast, the open number line encourages children to think about landmarks on a number line, to take the leaps mentally, and to visualize the landing points, rather than to simply read off the answer. Another limitation of the hundred chart is that it is not a linear representation of numbers as on a number line, but instead is analogous to prose, as read on a page. After 10 comes 11, but one must shift to the left and go down a line to find it. This can be a problem for children when they are developing a "number space"—a model to think with.

As you do minilessons from this resource unit, you will want to use models to depict children's strategies. Representing computation strategies with mathematical models provides children with images for discussion, and supports the development of the various strategies for computational fluency. The connecting cube train, the open number line, and money are the primary models used in this guide for addition and subtraction—assuming that the models have already been developed with realistic situations and rich investigations. In the *Contexts for Learning Mathematics* series, the units *Measuring for the Art Show* and *Ages and Timelines* can be used to develop the connecting cube and open number line models. The money model is developed in the *Contexts for Learning Mathematics* unit *The T-Shirt Factory*. If your students do not clearly understand the use of these models, you may find it beneficial to use the relevant units first. Once the model has been introduced as a representation of a realistic situation, you can use it to model the computation strategies that children explain.

A Few Words of Caution

As you work with the minilessons in this resource book, it is very important to remember two things. First, honor children's strategies. Accept alternative solutions and explore why they work. Use the models to represent children's strategies and facilitate discussion and reflection on the strategies shared. Sample classroom episodes (titled "Inside One Classroom") are interspersed throughout this resource guide to help you anticipate what learners might say and do, and to provide you with images of teachers and children at work. The intent is not to get all learners to use the same strategy at the end of the string. That would simply be discovery learning. The strings are crafted to support development, to encourage children to look to the numbers and to use a variety of strategies helpful for those numbers.

Second, do not use the string as a recipe that cannot be varied. You will need to be flexible. The strings are designed to encourage discussion and reflection on various strategies important for numeracy. Although the strings have been carefully crafted to support the development of these strategies, they are not foolproof: if the numbers in the string are not sufficient to produce the results intended, you will need to insert additional problems, depending on your students' responses, to provide more opportunities for learning. For this reason, most of the strings are accompanied by a Behind the Numbers section describing the string's purpose and how the numbers were chosen. Being aware of the purpose of each string will guide you in determining what type of problems to add. These sections should also be helpful in developing your ability to craft your own strings. Strings are fun both to do and to craft.

Resources

Dolk, Maarten, and Catherine Twomey Fosnot. 2004a. *Addition and Subtraction Minilessons, Grades PreK–3*. CD-ROM with accompanying facilitator's guide by Antonia Cameron, Sherrin B. Hersch, and Catherine Twomey Fosnot. Portsmouth, NH: Heinemann.

———. 2004b. *Fostering Children's Mathematical Development, Grades PreK–3: The Landscape of Learning*. CD-ROM with accompanying facilitator's guide by Sherrin B. Hersch, Antonia Cameron and Catherine Twomey Fosnot. Portsmouth, NH: Heinemann.